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BOTTLENECKS AND THE PHILLIPS CURVE: A DISAGGREGATED KEYNESIAN MODEL OF INFLATION, OUTPUT AND UNEMPLOYMENT*

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Benchmark macroeconomic models of output and inflation combine the demand side of the economy with aggregate supply equations based upon some version of the Phillips curve. In contrast to classical and new classical models such as Lucas (1972) and Sargent (1979, chs. I, XII and XVI), in which prices jump instantaneously to market clearing levels, the gradual price-adjustment approach represented by Tobin (1972), Laidler and Parkin (1975, section 4), Barro and Grossman (1976, ch. 5), Sargent (1979, ch. II and ch. V.I), Okun (1981), and Perry (1983) take wages and/or prices to be predetermined at an instant in time while their rates of change depend on the level of aggregate output.

In the prototype complete macroeconomic model incorporating this approach, output is determined by aggregate demand equations while the augmented Phillips curve specifies that prices move in a direction which will tend to return output to its equilibrium value. This short-run Keynesian, long-run monetarist character is typical of many structural macroeconometric models. It is the standard textbook account found, for example, in Dornbusch and Fischer (1981) and is implicit in much current research.

There are fundamental questions of consistency in the type of model just described. The central difficulty is that, with wages and prices predetermined, the non-market-clearing analysis of Barro and Grossman (1976, ch. 2) and Malinvaud (1977) is applicable. But in this world the aggregate demand equations are decisive only in the Keynesian regime: in the other regimes, which must arise at high levels of aggregate demand, the effective supplies of output or labour or both must be modelled.

Indeed, at the aggregate level it is difficult to provide a coherent account of how it is possible to have output above its equilibrium value since this would require labour and output in excess of their notional supplies, violating Barro and Grossman's 'min' condition that quantities be determined by the short side of the market. An appealing line for addressing this last difficulty is to disaggregate markets, an approach taken by Hansen (1970), Tobin (1972) and Barro and Grossman (1976, ch. 5). However, because each market can be in either excess supply or demand, a formal fixed-price general equilibrium model appears to be intractable, due to the resulting multiplicity of regimes, and is not attempted by these authors.

In this paper these problems are overcome in a disaggregated model in which

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the only departures from market clearing are temporary wage floors.[1] This approach has a venerable tradition discussed in Okun (1981, ch. 1). See also the rationales of Akerlof (1980) and Solow (1980). Incorporating and formalising several ideas due to Tobin (1972),[2] a dynamic general equilibrium model is developed which includes stochastic sectoral shocks as well as equilibrating movements of sectoral wage floors and labour supplies.

I. SPECIFICATION OF THE MODEL

I.1. *Demand for output*

Although we will develop a complete model of the economy, numerous simplifying assumptions will be made to facilitate aggregation and to focus attention on the central features of the model. Assume that the notional labour supply for each household is a fixed quantity. Either this amount or, if they are rationed, a smaller amount will actually be supplied. In either case we may take their nominal wage income and hence, since there will be zero profits, their total nominal income as given. Wealth is held in the form of outside money, the only asset in the model. Given their budget constraint of monetary wealth plus nominal income over the period, demand functions for real money balances and for each of the N goods can be derived from their preferences, which will be assumed to be identical across households.

The demand for real cash balances at time t is assumed to be governed by a transactions demand and to take the simple quantity theory form

$$m_t^d - p_t = q_t - v_t,$$

where m_t^d is the logarithm of money demand, v_t is the logarithm of the mean velocity per sector, assumed exogenous, and q_t and p_t are given by $q_t = N^{-1}\Sigma_{i=1}^N q_{it}$ and $p_t = N^{-1}\Sigma_{i=1}^N p_{it}$, where q_{it} is the logarithm of the quantity produced in sector i, for $i = 1, ..., N$, and p_{it} is the logarithm of the price level in that sector. Thus q_t and p_t, which we shall refer to as aggregate output and the aggregate price level, respectively, are the logarithms of the geometric means of output and prices in the N sectors. The use of equal weights in these fixed weight indices is a simplification which could be dropped without changing the analysis. We further assume that $v_t \sim \text{IIN}(v, \sigma_v^2)$, i.e. that v_t is independently and identically normally distributed with mean v and variance σ_v^2. Without loss of generality we can take $v = 0$. The logarithm of the money supply m_t is taken to be exogenous, and with equilibrium in the money market

$$m_t - p_t = q_t - v_t. \tag{1}$$

[1] An alternative way of attaining consistency is to drop the min condition, as done, for example, in Sargent's 'Keynesian model' in which employment is given by the demand for labour. Such models place no explicit upper bound on aggregate output and, as discussed by Hall (1980, p. 24), amount to a 'denial of the labour supply function' in the short run. In a static open economy model, Kennally (1983) assumes, as does Sargent in the Keynesian model, goods market clearing and sticky wages, but retains the min condition. Again, a disaggregated model along these lines appears to be intractable. For an empirical model of the labour market in which the min condition is imposed see Batchelor and Sheriff (1980).

[2] For an alternative model drawing on Tobin, see Iwai (1983).

Our assumptions effectively make exogenous a measure of nominal aggregate output.[1,2]

Assume that demand for the N produced goods is governed by a Cobb–Douglas utility function. When the money market is in equilibrium, nominal income becomes the effective budget constraint for the non-money goods and we may write the demand[3] for the output of sector i, $i = 1, \ldots, N$, as

$$q_{it} = q_t - (p_{it} - p_t) + d_{it}, \tag{2}$$

where by aggregation we have $N^{-1}\Sigma_{i=1}^{N} d_{it} = 0$. The specification of unit price and income elasticities, implied by the Cobb–Douglas utility function, could be relaxed in favour of a more general demand system, but substantially simplifies the presentation of the model. d_{it} is a taste parameter that is predetermined at time t but shifts randomly over time. In particular,

$$d_{it} = d_{i,t-1} + u_{it}^{d} \quad \text{where} \quad u_{it}^{d} \sim \text{IIN}(0, \sigma_d^2), \tag{3}$$

so that in each sector the demand intercept shifts according to a random walk. We also assume that at each time the u_{it}^{d} are 'nearly' independent across sectors. Because of the constraint $\Sigma_i d_{it} = 0$ there is a negative correlation of order N^{-1} which we assume is sufficiently small to ignore.

1.2. *Firms*

Firms in each sector produce output using only one input, a type of labour specific to that sector, under conditions of constant returns to scale. If k_i is the logarithm of the unit labour requirement in sector i, assumed fixed over time, and if n_{it} is the logarithm of the quantity of labour of type i hired in sector i at time t, then $q_{it} = n_{it} - k_i$. Firms produce under conditions of perfect competition, hiring labour at a wage rate the logarithm of which is x_{it} and selling output at a price determined by the zero profit condition $p_{it} = x_{it} + k_i$.

Note that output prices move flexibly to clear the market. Because of constant returns to scale, the supply curve of the firm is perfectly elastic with output determined by demand.[4]

1.3. *The Labour Market*

At each moment every worker is located in a specific labour market and the logarithm of the labour supply in sector i is l_{it}. However, there is a base wage, the logarithm of which is w_{it}, which acts as a floor below which the actual wage cannot fall and consequently actual employment, the logarithm of which is n_{it}, may fall short of labour supply. The wage in each sector otherwise moves flexibly, so that in each sector there are two possible states, depending on labour supply

[1] This may also be thought of as the special case of IS–LM in which the LM curve is vertical. We choose a simple representation of aggregate demand in order to focus on aggregate supply.

[2] Here the logarithm of money demand depends on the logarithm of the geometric mean of the value of sectoral outputs which, of course, is not always identical with the logarithm of the arithmetic mean. This choice simplifies the algebra.

[3] Equation (2) would not hold out of money market equilibrium since a real balance effect would need to be included. For a discussion of this point see Patinkin (1965, chs. II–IV, VIII).

[4] Although chosen for algebraic convenience, these assumptions do have the advantage of being consistent with the stylised fact that real wages are largely independent of the level of demand.

and derived demand: (i) a bottleneck (or market clearing) state with $n_{it} = l_{it}$ and $x_{it} \geqslant w_{it}$ and (ii) an excess supply state with $n_{it} < l_{it}$ and $x_{it} = w_{it}$. The two cases are illustrated in Fig. 1 by points *a* and *b* respectively.[1] The use of the term 'bottleneck' for the state in which there is no excess supply is somewhat unusual (because of the market clearing) but is intuitive and follows Keynes (1936, ch. 21).

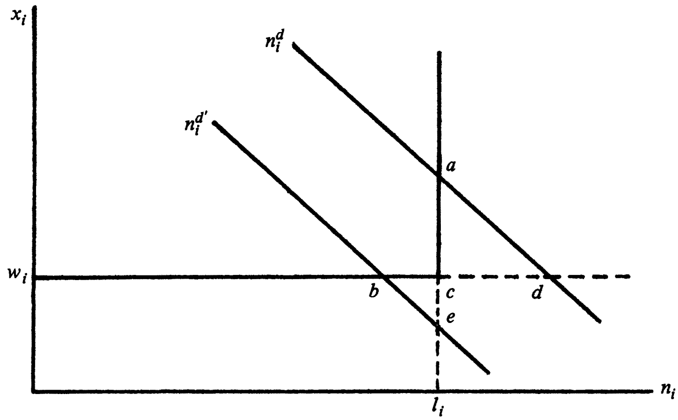

Fig. 1. Labour market in sector *i*. *Note*: w_i is the base wage or wage floor. n_i^d and n_i^d are two different possible derived demand curves for sector *i*, with equilibria *a* and *b* respectively. With demand n_i^d the excess demand for labour D_i that would obtain at the base wage is the length of segment *cd* and the market clearing wage is the height of point *a*. With demand n_i^d the excess supply of labour $-D_i'$ is the length of *bc*, and the wage that would be necessary to clear the market is the height of point *e*.

The remainder of the model specifies the movement of sectoral base wages and labour supplies over time. Base wages are assumed to adjust according to augmented Phillips curves:

$$\Delta w_{it} = \xi D_{i,t-1} + z_t + u_{it}^a + \Delta e_t, \tag{4}$$

where $\Delta w_{it} = w_{it} - w_{i,t-1}$ and D_i is excess demand for labour of type *i*, defined below. We assume $0 \leqslant \xi \leqslant 1$ which, given the unit elastic demand for labour of type *i*, rules out over-adjustment of the base wage in response to excess demand. z_t is inflationary momentum, $u_{it}^a \sim \text{IIN}(0, \sigma_a^2)$ is a white-noise shock to the base wage in sector *i* assumed 'nearly' independent across sectors and $e_t \sim \text{IIN}(0, \sigma_e^2)$ is a white-noise shock to the average base wage, chosen to enter in first-difference form in order to avoid persistent drifting of the average base wage away from its mean path.[2]

D_{it} is defined as the difference between the derived demand for labour in sector *i*, computed at $x_{it} = w_{it}$, and the labour supply l_{it}, and is given by

$$D_{it} = p_t + q_t - w_{it} - l_{it} + d_{it}. \tag{5}$$

[1] The assumption of inelastic labour supplies, l_i, is made for algebraic convenience, and relaxing this assumption would not alter the qualitative results. In particular the vertical aggregate supply curve when $b = 1$ derived in Section II would be unaffected, since this depends instead on the neutrality of money when all markets clear.

[2] It would also be possible to include in the model a general wage floor which is independent of demand and is determined by non-employment sources of income.

For the economy as a whole we have $D_t = p_t + q_t - w_t - l_t$, where variables without an i subscript indicate the average value across the N sectors. In equation (4) we have assumed that Δw_{it} depends on the lagged value of D_i. This formulation makes w_{it} predetermined up to exogenous white noise and should be thought of as the discrete time approximation to a differential equation in which dw_{it}/dt depends on contemporaneous D_i.

Inflationary momentum is assumed to adapt to the actual rate of growth of wages:

$$z_t = \beta \Delta x_{t-1} + (1-\beta) z_{t-1}, \quad 0 < \beta \leqslant 1. \tag{6}$$

A more general formulation would allow z to include forward- as well as backward-looking components along the lines of Taylor (1979). The importance of a large backward-looking component to z in the non-market clearing approach has been stressed by Okun (1981, ch. VI) and Perry (1983). Analytical difficulties resulting from incorporating forward-looking rational expectations in models with aggregate non-linearities are discussed in Taylor (1983), and it is unlikely that allowing z to have mixed components would alter the major qualitative results of our model.

Labour supply in each sector, while assumed to be predetermined at each time, moves between sectors over time depending on their expected relative attractiveness. In particular, it is assumed that the net labour flow into each sector depends on its 'shadow relative wage', the difference between that sector's market clearing wage and the economy-wide average. Depending on the base wage in that sector, a positive shadow relative wage may take the form of a higher than average actual wage, a lower than average unemployment rate or both.[1] Let $\theta_{it} = l_{it} - l$ measure the relative labour supply in sector i, where for convenience we assume that the aggregate labour supply is constant over time so that $l_t = l$. Then the market clearing wage determined by $n_{it} = l_{it}$ would be

$$p_t + q_t - l + d_{it} - \theta_{it}$$

and the shadow relative wage is $d_{it} - \theta_{it}$. Hence we assume

$$\Delta \theta_{it} = \lambda \mathrm{E}_{t-1}(d_{it} - \theta_{it}) + u^{\theta}_{it}, \quad 0 < \lambda < 1, \tag{7}$$

where u^{θ}_{it} is a white-noise shock to labour supply[2] in sector i, assumed 'nearly' independent across sectors, and E_{t-1} denotes the mathematical expectation conditional on information available at time $t-1$. Underlying equation (7) is the assumption that there are costs of moving or changing job, with λ varying inversely with some measure of these costs.[3]

[1] In general, the net flow of labour to a sector would also depend on the rationing scheme for labour, in particular the likelihood of newcomers to the market obtaining a job compared to the likelihood of those already employed retaining their job.

[2] We also assume that u^{a}_{it}, u^{θ}_{it} and u^{d}_{it} are independent of each other at all times.

[3] This assumption is similar to the treatment of migration by Harris and Todaro (1970).

II. SHORT-RUN EQUILIBRIUM

At each point in time we can take as given the exogenous or predetermined variables

$$S_t = \{m_t, v_t, [(d_{it}, l_{it}, w_{it}, k_i), i = 1, \ldots, N]\}.$$

'Short run' equilibrium values can then be calculated for

$$(p_{it}, x_{it}, q_{it}, n_{it}), \quad i = 1, \ldots, N,$$

as well as their aggregate (average) values p_t, x_t, q_t and n_t. Holding all variables in S_t constant except m_t we can compute the values of the endogenous variables as functions of m_t and in particular generate a correspondence between p_t and q_t which we will refer to as the aggregate supply curve. In some respects the term 'aggregate supply' is a misnomer since, as will become apparent, it depends crucially on sectoral demand parameters. However, we will retain the term since, given these and other sectoral parameters, the aggregate supply curve traces out the combinations of aggregate price and output resulting from variations in aggregate demand.

For notational simplicity we omit time subscripts for the remainder of this section. The dependence of p and q on S is mediated by certain key variables. For sector i let $r_i = d_i - a_i - \theta_i$, where $a_i = w_i - w$ is the base wage differential. Note that $\Sigma_i r_i = 0$. We will refer to the set $R = \{r_1, \ldots, r_N\}$ as the distribution of bottlenecks (actual or potential) since it describes the extent of imbalance between sectors. Indeed it follows from (5) that $D_i = D + r_i$, so that $\{r_1, \ldots, r_N\}$ determines the order in which sectors enter the bottleneck state, high values of r_i entering first, as aggregate demand D is increased. Since $D = m + v - w - l$ we have $R = R(S)$ and $D = D(S)$. Let $0 \leqslant b \leqslant 1$ be the proportion of sectors experiencing bottlenecks, i.e. $b = N^{-1}\Sigma_i I_i$, where $I_i = 1$ if $D_i \geqslant 0$ and $I_i = 0$ if $D_i < 0$. Then also $b = b(S)$.

We state the short-run results as a pair of Propositions. The proofs of all propositions are to be found in the Appendix.

PROPOSITION 1. *In short-run equilibrium we have*

$$p = w + k + \tilde{p}(R, D), \quad q = l - k + \tilde{q}(R, D) \quad \textit{and} \quad b = b(R, D),$$

where $\tilde{p}(R, D)$, $\tilde{q}(R, D)$ and $b(R, D)$ are non-decreasing functions of D. Furthermore $\tilde{p}$ and $\tilde{q}$ are continuous, piecewise linear, and right-hand-side differentiable with respect to D, with

$$\frac{\partial \tilde{p}}{\partial D} = b \quad \textit{and} \quad \frac{\partial \tilde{q}}{\partial D} = 1 - b.$$

Proposition 1 states that, given the distribution of possible bottlenecks, the aggregate price and output levels are continuous non-decreasing functions of the aggregate excess demand for labour, or equivalently, of the money supply. The proportions in which a change in the money supply divides between price and output are given by

$$\frac{\partial p}{\partial m} = b \quad \text{and} \quad \frac{\partial q}{\partial m} = 1 - b.$$

PROPOSITION 2. A. *Holding R fixed, the proportion of bottlenecks is an implicit function of* $q, b(q)$, *defined for* $q \leqslant l-k$, *such that* $b(q)$ *is non-decreasing, with* $b(q) = 0$ *for* $q \leqslant l-k-r^*$, *where* $r^* = \max_i r_i$, *and* $b(q) = 1$ *for* $q = l-k$.

B. *Holding R fixed, the aggregate supply schedule can be written*

$$p(q) = w+k+f(q-l+k) \tag{8}$$

for $q < l-k$, *and* $p \geqslant w+k-r_*$ *for* $q = l-k$. f *is a continuous, non-decreasing piecewise linear, convex function defined for* $q \leqslant l-k$, *with* $f(q-l+k) = 0$ *for* $q \leqslant l-k-r^*$, *and* $f(q-l+k) = -r_*$ *for* $q = l-k$, *where* $r_* = \min_i r_i$. *Furthermore,*

$$\frac{dp}{dq} = b(1-b)^{-1}. \tag{9}$$

Proposition 2 solves for p in terms of q yielding the non-decreasing convex aggregate supply curve (8) sketched in Fig. 2. Its detailed shape depends on the

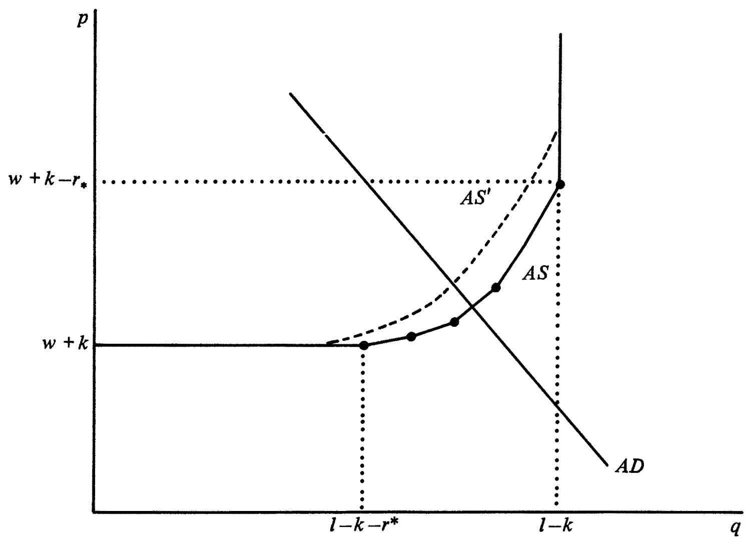

Fig. 2. Determination of short-run equilibrium. *Note*: *AS* is the aggregate supply curve and *AD* is the aggregate demand curve. *AS′* is an aggregate supply curve with a greater dispersion in the distribution of bottlenecks.

distribution of possible bottlenecks, R. The short-run equilibrium levels of q and p are determined by the intersection of this aggregate supply curve with the aggregate demand curve $p = m+v-q$. The macroeconomic state is captured by b, and the cases $b = 0$ and $b = 1$ can be thought of as Keynesian and classical polar cases. A greater degree of imbalance between sectors will shift the aggregate supply curve (for $0 < b < 1$) up and to the left, as indicated by the curve AS' in Fig. 2. This is straightforward to show if there is a proportional worsening in the distribution of bottlenecks from $R = \{r_i\}$ to $R' = \{r_i'\}$ where $r_i' = \mu r_i, \mu > 1$, for $i = 1, ..., N$.

The model of short-run equilibrium described in this section formalises ideas found in Keynes (1936, ch. 21), particularly in the emphasis on the importance of bottlenecks (interpreted in this paper as labour bottlenecks).[1] This account differs from other derivations of the aggregate supply curve in principle as well as precision. Some alternative descriptions rely on diminishing marginal returns to labour; in contrast our model assumes constant returns to labour in each sector. Other derivations are based on an artifact resulting from the inclusion of current demand in a discrete-time Phillips equation and would disappear in a continuous-time formulation. Both of these effects have been omitted in this model in order to focus attention on the 'bottlenecks' approach which, in contrast to them, resolves the theoretical difficulties described in the introduction.

III. EQUILIBRIUM IN THE LONG RUN

Suppose that monetary authorities expand the money supply according to the rule $\Delta m_t = g$, where g is a constant proportional rate of growth. We now investigate the dynamics of the model and show that a stochastic equilibrium is reached.

We begin with the distribution of bottlenecks $R = \{r_1, \ldots, r_N\}$, which describes the degree of imbalance between sectors. Since $r_{it} = d_{it} - \theta_{it} - a_{it}$, R depends on sectoral differences in output demands, labour supplies and base wages. Random shocks to each of these factors tend to increase the dispersion of bottlenecks, but tending to offset them are equilibrating movements of relative base wage rates and labour supplies. At any particular moment R is predetermined, but we can look for an equilibrium distribution. From (4) it follows that relative base wages $a_{it} = w_{it} - w_t$ change according to

$$\Delta a_{it} = \xi r_{i,t-1} + u_{it}^a. \tag{10}$$

Relative labour supplies θ_{it} move in accordance with (7) and changes in sectoral demand are given by (3). It is apparent that d_{it}, θ_{it} and a_{it}, and hence R_t, follow stochastic processes which are exogenous to the rest of the model, and it can be shown that R_t approaches a stationary distribution in the long run.

PROPOSITION 3. *For each $i = 1, \ldots, N$, r_{it} is an* ARMA(2, 1) *process which approaches a limiting stationary distribution with variance*

$$\sigma_r^2 = \frac{2(\sigma_d^2 + \sigma_\theta^2)}{(2-\xi)(2-\lambda')[1-(1-\xi)(1-\lambda')]} + \frac{\sigma_a^2}{\xi(2-\xi)}, \tag{11}$$

where $\lambda' = \lambda/(1+\lambda)$. Let r_t be the empirical distribution of $r_{1t}, \ldots, r_{Nt}$. Then for large N, as $t \to \infty$, r is approximately given by the limiting distribution $r \sim N(0, \sigma_r^2)$.

From (11) it follows that increases in σ_d^2, σ_θ^2 and σ_a^2 increase σ_r^2 and, provided $\xi < \frac{1}{2}$ so that the discrete approximation is valid, increases in ξ and λ decrease σ_r^2.

This equilibrium distribution of bottlenecks determines an aggregate supply curve given in the following proposition.

PROPOSITION 4. *The long-run equilibrium aggregate supply curve is given implicitly by*

$$p = u + k + (p+q-w-l)\,\Phi\left(\frac{p+q-w-l}{\sigma_r}\right) + \sigma_r \phi\left(\frac{p+q-w-l}{\sigma_r}\right), \tag{12}$$

[1] The passage headed (3) beginning on p. 300 of the *General Theory* makes the connection particularly clear.

where Φ is the cumulative distribution function of the standard normal and ϕ is the density function of the standard normal.

Finally, this equilibrium aggregate supply curve can be combined with the aggregate Phillips curve

$$\Delta w_t = \xi D_{t-1} + z_t + \Delta e_t, \tag{13}$$

obtained by averaging (4) across sectors, and the aggregate-demand equations to determine the equilibrium level of output. The corresponding equilibrium level of employment is determined by $n = q + k$, and from this can be computed the natural rate of unemployment, defined as one minus the ratio of equilibrium employment to labour supply.

PROPOSITION 5. *Consider the behaviour of aggregate output and the inflation rate for large t. q_t approximately follows a stationary* ARMA(2, 2) *process, for small σ_v^2 and σ_e^2, with a mean value of $\bar{q} = \mathrm{E}(q_t) = l - k - \sigma_r/\surd(2\pi)$. The corresponding natural rate of unemployment is given by*

$$\bar{U} = 1 - \exp[-\sigma_r/\surd(2\pi)], \tag{14}$$

and the corresponding mean rate of inflation is $\mathrm{E}(\Delta p_t) = g$.

Since the long-run mean level of output is independent of the rate of growth of the money supply it is appropriately referred to as the 'natural rate of output'. The determination of $\bar{q}$ is shown geometrically in Fig. 3. There

$$p = w + k + f(q - l + k)$$

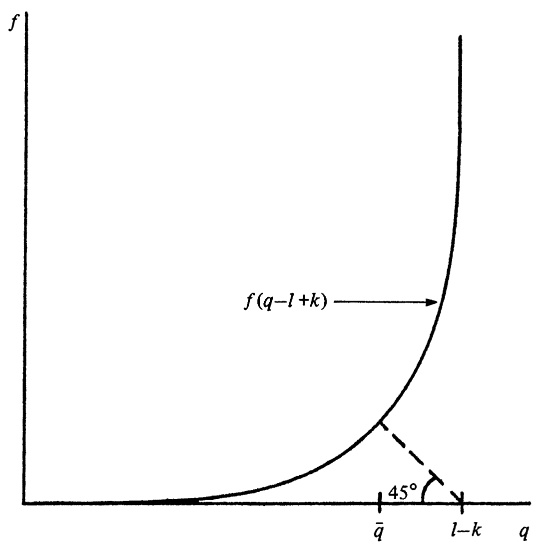

Fig. 3. Determination of natural rate of output.

represents the aggregate supply curve given implicitly in (12). Intuitively, $\bar{q}$ is such that the upward pressures on the change of base wages in the bottleneck sectors are just offset by the downward pressures in sectors with unemployment, and indeed $\bar{q}$ corresponds to $D = 0$, i.e. to aggregate balance in the labour markets.

A positive natural rate of unemployment arises because of equilibrium structural imbalance. In each labour market experiencing unemployment, the unemployment is involuntary in the sense that at the prevailing wage in that sector there is an excess supply of labour. Although labour moves between sectors, the associated costs have been assumed to be such that instantaneous adjustments do not take place. Consequently, imbalances between sectors, sustained by random shocks despite equilibrating forces, result in unemployment at levels of aggregate demand associated with constant rates of inflation in the long run.

IV. MEDIUM-RUN DYNAMICS

Over the medium run the responses of q_t and Δp_t to exogenous changes can be complex and depend on the relative sizes of key parameters. We consider the special case, which many believe to be of empirical importance, in which ξ is sufficiently small to ignore except in the long run. We assume that the distribution of bottlenecks is in equilibrium and consider the effects on inflation of changes in output achieved through monetary policy.

The dynamics of this case can be seen most easily by developing a single supply-side equation for Δp_t. Combining (8), (13), (6) and $p_t = x_t + k$, and substituting in recursively, we obtain

$$\Delta p_t = c + f(q_t - l + k) - (1-\beta) f(q_{t-1} - l + k) + e_t - (1-\beta)\, e_{t-1}, \tag{15}$$

where c depends on initial conditions and other factors which will change over the long run but which may be neglected over the medium run provided ξ is small.[1] If by manipulation of monetary policy we were able to hold output fixed at $q_t = q$ then the expected value of inflation would be given by

$$\mathrm{E}(\Delta p_t) = c + \beta f(q - l + k). \tag{16}$$

We thus obtain a medium-run trade-off between output and inflation based not on the usual Phillips curve effect (which is being ignored under our assumption that ξ is close to 0) but on a dynamic version of the bottleneck phenomenon of Section II. An increase in q raises Δp_t due to price increases in bottleneck sectors, and in subsequent periods this higher inflation is incorporated into inflationary momentum and hence spread throughout the economy in a process similar to the sectoral-shift model of Schultze (1959). By Proposition 2 the slope of the medium-run trade-off is $\beta b(1-b)^{-1}$.

The medium-run dynamics of the small ξ case provide a caution to policy makers. A recession would initially bring a substantial reduction in inflation accompanying the drop in aggregate output, but may have a disappointingly small effect on inflation near the trough, when both the rate of change of output and the proportion of sectors in bottlenecks are small. A recovery could increase inflation even when output is below its natural rate.

[1] c is given by

$$c = \Delta p_0 - \beta[f(q_0 - l + k) + e_0] - (1-\beta)\,[\Delta f(q_0 - l + k) + \Delta e_0] + \xi(D_{t-1} - D_{-1}) + \xi\beta \sum_{s=0}^{t-1} D_{s-1}.$$

We also note that Gordon's 'Rate of Change' equations (1980) can be derived from our medium-run model. Gordon has shown that good empirical explanations of inflation in many countries are obtained by regressions of Δp_t on distributed lags of itself and on the rate of change of nominal output,

$$\Delta Y_t = \Delta p_t + \Delta q_t.$$

Taking the first difference of (15) and making the linear approximation

$$\Delta f(q_t - l + k) \approx b(1-b)^{-1} \Delta q_t$$

one can obtain

$$\Delta p_t = \frac{\beta(1-b)}{1-(1-\beta)\mathrm{L}} \Delta p_{t-1} + b\Delta Y_t + (1-b)\,\Delta e_t, \tag{17}$$

where L is the lag operator. In addition to providing a rationale for a 'rate of change' effect, equation (17) implies an important refinement. Since the coefficient on ΔY_t is the proportion of sectors experiencing bottlenecks, the effect should be strongest at high levels of real output relative to trend.

Other interesting medium-run dynamics would result from a perturbation of the distribution of bottlenecks away from its equilibrium. Major exogenous shifts in relative demands for goods or sectoral labour supplies are likely to affect the time paths of output and inflation adversely. The framework of this paper may thus provide convenient interpretations for the Phillips curve shifts attributed by Perry (1970) to the changing composition of the labour force and for the recent findings of Lilien (1982) that unemployment can be largely explained empirically by the sectoral dispersion in rates of growth of output.

V. CONCLUSIONS

Standard macroeconomic models with predetermined prices, combining aggregate demand-determined output with an augmented Phillips curve, encounter difficulties as a result of failing to incorporate binding effective supplies at output levels above the natural rate. This problem can be overcome in a disaggregated model in which it is assumed in each sector that prices are flexible and that wages are flexible subject to their not falling below a predetermined floor which adjusts endogenously over time. A dynamic general equilibrium model is developed, allowing for stochastic sectoral shocks to demand, wages and labour supply, as well as equilibrating movements of sectoral wage floors and labour flows. The resulting model stresses the importance of bottlenecks and structural imbalances in determining the short-run aggregate supply curve, the long-run natural rate of unemployment and the dynamics of output and inflation.

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Appendix

Proof of Proposition 1. From (5), (2) and 1.2 it follows that $D_i = (n_i - l_i) + (x_i - w_i)$. Since $x_i \geqslant w_i$ and $n_i \leqslant l_i$, with complementary slackness,

$$\tilde{q}_i = \begin{cases} 0 & \text{if } D_i \geqslant 0, \\ D_i & \text{if } D_i < 0 \end{cases} \quad \text{and} \quad \tilde{p}_i = \begin{cases} D_i & \text{if } D_i \geqslant 0, \\ 0 & \text{if } D_i < 0, \end{cases}$$

where $\tilde{q}_i = q_i - l_i + k_i$ and $\tilde{p}_i = p_i - w_i - k_i$. Substituting $D_i = D + r_i$ and averaging over the N sectors we get

$$\tilde{p} = bD + \frac{1}{N} \sum_{D+r_i \geqslant 0} r_i \quad \text{and} \quad \tilde{q} = (1-b)\, D + \frac{1}{N} \sum_{D+r_i<0} r_i.$$

Proof of Proposition 2. This follows from Proposition 1, and its proof, and from the inverse function theorem.

Proof of Proposition 3. From (7) we have $\Delta\theta_{it} = \lambda'(d_{i,t-1} - \theta_{i,t-1}) + u^{\theta}_{it}$, where $\lambda' = \lambda(1+\lambda)^{-1}$. Then, using (3) and (10),

$$[1-(1-\xi)\,L]\,[1-(1-\lambda')\,L]\,r_{it} = \Delta u^d_{it} - \Delta u^{\theta}_{it} - u^a_{it} + (1-\lambda')\,u^a_{i,\,t-1},$$

where L is the lag operator. Since $0 < \xi$, $\lambda' < 1$, r_{it} approaches a stationary ARMA (2, 1) process. (11) is obtained by computing σ^2_r in the usual way for ARMA processes. The r_{it} are normal and 'nearly' independent across i, so their empirical distribution for large N is $r \sim N(0, \sigma^2_r)$.

Proof of Proposition 4. Following the line of argument to derive the expression for $\tilde{p}$ in Proposition 1 we have

$$\tilde{p} = bD + \int_{-D}^{\infty} rg(r)\, dr,$$

where g is the density of the normal distribution with mean 0 and variance σ^2_r. Since $D = \tilde{p} + \tilde{q}$ it follows that

$$b = \Phi\left(\frac{\tilde{p}+\tilde{q}}{\sigma_r}\right).$$

Making these substitutions and performing the integration we obtain (12).

Proof of Proposition 5. Combining (13), (1), (5), (6), (8) and $p_t = x_t + k$ and linearising around $\bar{q} = \mathrm{E}(q_t)$ we obtain

$$\begin{aligned} q_t - (2-\xi-b\beta)\, q_{t-1} + [1-\xi(1-\beta)-b\beta]\, q_{t-2} \\ = \beta\xi b\bar{q} + \beta\xi(1-b)\,[l-k-f(\bar{q}-l+k)] \\ + (1-b)\,[\Delta v_t - \Delta e_t - \Delta v_{t-1} + (1-\beta)\,\Delta e_{t-1}]. \end{aligned}$$

Applying the Schur Theorem it follows that q_t tends to a stationary process. Taking expectations of both sides we have $\bar{q} + f(\bar{q}-l+k) = l-k$ provided the variances of v_t and e_t are small.[1] Hence $p + \bar{q} - w - l = 0$, which with (12) implies $\bar{q} = l - k - \sigma_r/\sqrt{(2\pi)}$, from which (14) follows from the definition of U.

[1] When σ^2_v and σ^2_e are not small, we cannot ignore quadratic terms in q_t when making the Taylor's series approximation of $f(q_t)$. This would lead to terms of the form f'' var (q_t) in the expression for $\mathrm{E}(q_t)$ resulting from taking expectations of the q_t equation. var (q_t) in turn would depend on σ^2_e and σ^2_v.

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