Supply, Demand, and Specialized Production

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Presentation for Better Policy Project

June 16, 2021

- Many economists believe demand shocks are a key driver of business cycles
  - Mian and Sufi (*Ecmta* 2014); Michaillat and Saez (*QJE* 2015); Auerbach, Gorodnichenko and Murphy (2020)
- This paper: demand shocks can cause recession even with perfectly flexible prices

- Related papers
  - Hamilton (*JPE* 1988); Angeletos (*JEEA* 2018); Angeletos and Lian (2020); Ilut and Saijo (*JME* 2021)
  - Murphy (*EER* 2017) and Auerbach, Gorodnichenko, and Murphy (2020) emphasize capacity constraints (exogenous in their models) and near-zero marginal costs
- This paper: capacity and marginal costs determined endogenously in equilibrium growth model

- Key technological friction: some goods can only be produced using specialized resources committed in advance
- This paper: labor is only factor of production
  - Production of some goods requires training and assembling a team of workers dedicated to producing that particular good
- If successful, the unit has a monopoly in producing that specialized good and maximizes profit subject to the unit's capacity

- If demand for good *j* falls, profit maximization calls for lowering both quantity and relative price
- Price does not fall more than this because it would mean lower profits
- No market force to bid costs down since marginal production cost is zero
- No offsetting gain from higher relative demand for other goods because productive resources cannot costlessly shift

- Key state variable: n_{1t} = fraction of population without a high-skill, high-paying job
- Value of n_{1t} is determined endogenously as individuals compare costs and benefits of specialization (predetermined at t)
- A sufficiently large drop in demand for good j may induce team to stop producing and join pool of unskilled

- But n_{1t} is also a factor in demand for all goods, because unskilled have lower income
- Keynesian-type multiplier: a decrease in demand for one good can lead to an overall drop in demand for all goods

Numerical example

- Economy starts out at t_0 with all variables on steady-state growth path except that demand is 10% lower than normal for 15% of goods
- In absence of general equilibrium feedback, impacted goods lower production and price by 10% (or 1.5% of GDP)
- Low demand persists for 5 periods and then returns to steady state
- Everyone knows all this at t_0 and base all decisions on perfect foresight

Fraction of workers without skills
0.03
0.02
0.01
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14
16
18
Real GDP
0.01
0
-0.01
-0.02
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16
18

Contributions of the paper

(1) Asymmetry: a decrease in demand changes output more than an increase in demand

- Empirical evidence: Weise (*JMCB* 1999); Lo and Piger (*JMCB* 2005)
- Tobin (*AER* 1973) and Ball and Mankiw (*Econ J* 1994) attributed to mechanics of price adjustment
- Here results from technology:
  - Demand below capacity leads to price and quantity decrease
  - Demand above capacity leads only to price increase

Contributions of the paper

(2) Humped-shape response to demand shocks

– Maximum effect on output can come many months after shock

  - Christiano, Eichenbaum and Evans (*JPE* 2005); Hamilton (*JME* 2008); Auclert, Rognlie and Straub (2020)

– Here arises from multiplier effects of n_{1t}

Contributions of paper

(3) Explains steady unemployment as equilibrium outcome in a growing economy

– Martellini and Menzio (*JPE* 2020)

(4) Unified model of growth and fluctuations

– Demand and other variables contribute to short-run fluctuations

– Long-run growth determined solely by productivity and population growth

Contributions of paper

(5) New model of income inequality as arising from successful gambles to create new goods

(6) Shows how monopoly power can be sustained in a growing economy even as new goods are introduced and some old goods discontinued every period

(7) Considerable heterogeneity across goods and individuals, but both individual and aggregate outcomes can be calculated using only a handful of equations

Limitations of paper

(1) At t_0 everyone knew demand levels for t_0, t_0+1,…
  - But they did not know this was going to happen in $t_0 - 1$ ("MIT shocks")
  - Examining outcome of realization of event that ex ante had very low probability

Limitations of paper

(2) Goods nonstorable, no capital, no financial markets, no intermediate goods

- Only dynamic programming problems are decision to try to develop a skill and decision of existing goods to continue production
- By focusing on this single source of specialization and technological friction, illuminate interaction between specialization and demand as determinant of short-run level of GDP

2. Demand for goods

At time t a set $j \in \mathcal{J}_t$ different goods are produced

Individual i purchases q_{ijt} of good j

$$\max \sum_{j \in \mathcal{J}_t} \frac{-\gamma_{ijt}}{2} (\bar{q}_{ijt} - q_{ijt})^2 \text{ s.t. } \sum_{j \in \mathcal{J}_t} P_{jt} q_{ijt} \leq y_{it}$$

Advantages of quadratic preferences:

- changing elasticity along demand curve (Murphy 2017)
- if price is too high, quantity demanded = 0

Disadvantages of quadratic preferences:

- do not make sense is q_{ijt} is growing with $\bar{q}_{ijt}$ fixed

Individual utility and demand curves

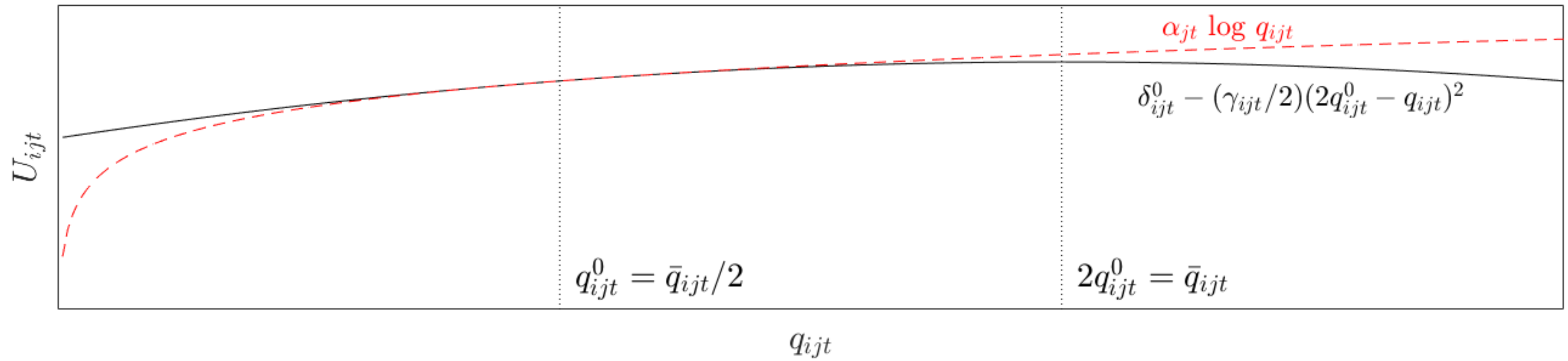
Solution:

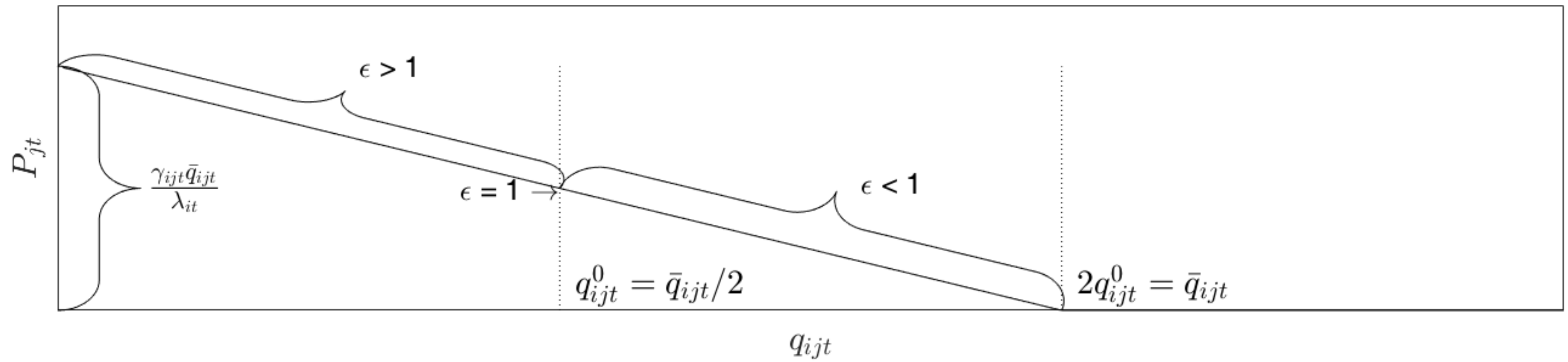
let $\bar{q}_{ijt}$ grow along the steady-state growth path for individual i

$$\sum_{j \in \mathcal{J}_t} \alpha_{jt} \log q_{ijt} \simeq \sum_{j \in \mathcal{J}_t} \left[\delta^0_{ijt} - \frac{\gamma_{ijt}}{2} (\bar{q}_{ijt} - q_{ijt})^2 \right]$$

$$\bar{q}_{ijt} = 2q^0_{ijt}$$

$$\gamma_{ijt} = \alpha_{jt} / (q^0_{ijt})^2$$


α_{jt} log q_{ijt}
$\delta^0_{ijt} - (\gamma_{ijt}/2)(2q^0_{ijt} - q_{ijt})^2$
U_{ijt}
$q^0_{ijt} = \bar{q}_{ijt}/2$
$2q^0_{ijt} = \bar{q}_{ijt}$
q_{ijt}


$\epsilon > 1$
P_{jt}
$\frac{\gamma_{ijt}\bar{q}_{ijt}}{\lambda_{it}}$
$\epsilon = 1$
$\epsilon < 1$
$q^0_{ijt} = \bar{q}_{ijt}/2$
$2q^0_{ijt} = \bar{q}_{ijt}$
q_{ijt}

Model a shock to demand for good j as $\bar{q}_{ijt} = \chi_{jt} 2 q_{ijt}^0$

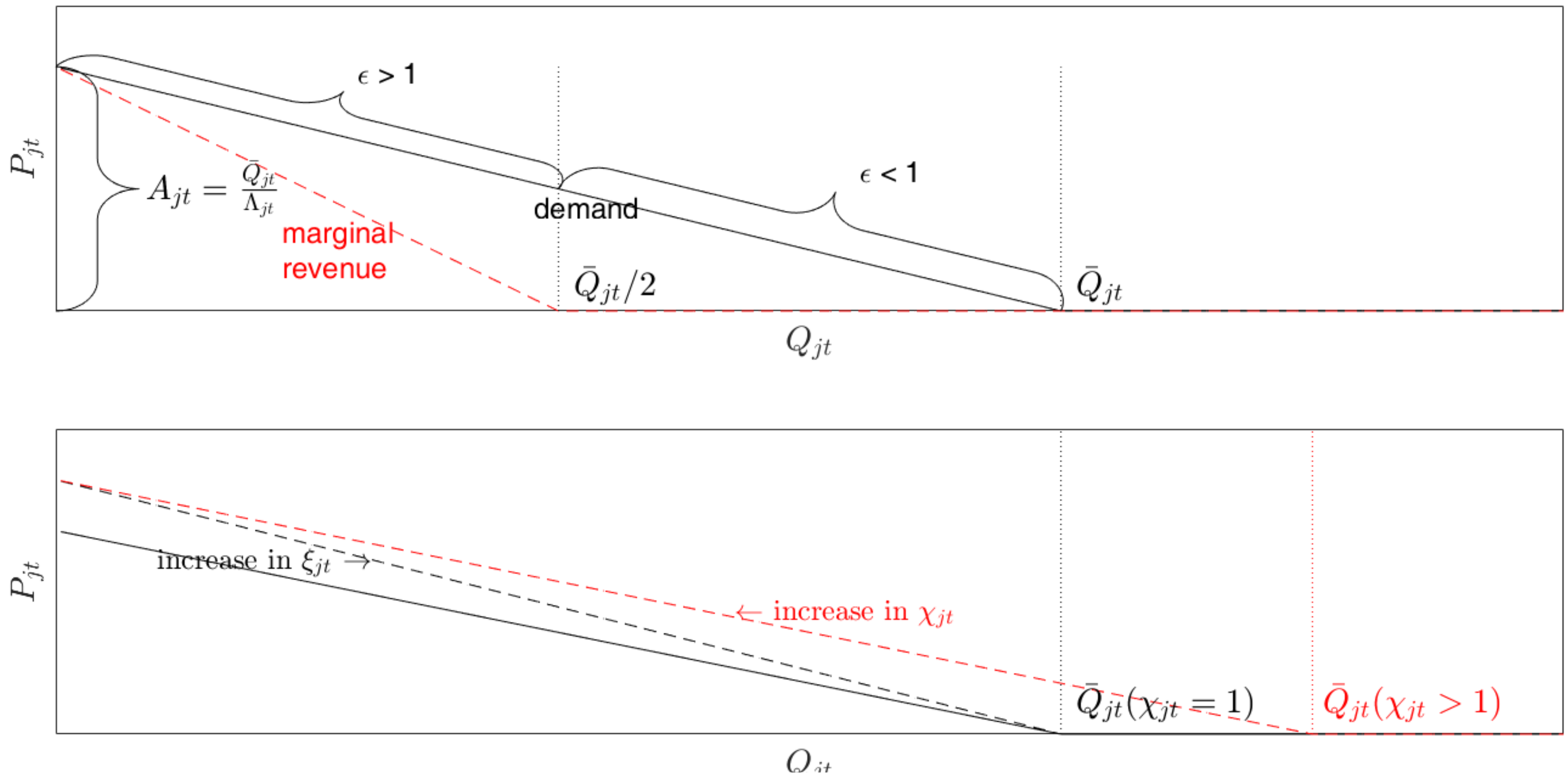
Along steady-state growth path, $\chi_{jt} = 1$ and each consumer spends a fraction α_{jt} of income on good j

$\alpha_{1t} = \alpha_1$ (= 40%)

Total demand for good *j*

$$Q_{jt} = \int_0^{N_t} q_{ijt} di$$

$$P_{jt} = A_{jt} - B_{jt} Q_{jt}$$



3. Production of specialized goods

Good $j = 1$ can be produced by anyone

(no training, competitive market)

Goods $j > 1$ require a dedicated team

N_{jt} = measure of team (determined in $t - 1$)

X_{jt} = productivity per person (exogenous)

$X_{jt}N_{jt}$ = capacity

$Q_{jt} = \min\{X_{jt}N_{jt}, \bar{Q}_{jt}/2\}$

$Y_{jt} = P_{jt}Q_{jt}/N_{jt}$

$Q_{jt} = \min\{X_{jt}N_{jt}, \bar{Q}_{jt}/2\}$

A demand shock multiplies $\bar{Q}_{jt}$ by χ_{jt}

A productivity shock multiplies X_{jt} by ζ_{jt}

Team could not be productive if loses any member

Operates as consortium, sharing profits equally

Determination of $N_{j,t+1}$

Team could add new workers who undergo training
Adds workers to maximize $t+1$ expected revenue
$N^*_{j,t+1} E_t(X_{j,t+1}) = E_t(\bar{Q}_{j,t+1}/2)$
$O_{jt} = \max\{N^*_{j,t+1} - N_{jt}, 0\}$

4. Unskilled workers

An unskilled individual i chooses between 3 options

goal: $v_{it} = \max \sum_{s=0}^{\infty} \beta^s E_t \log y_{i,t+s}$

Option 1: produce good 1

$y_{it} = P_{1t} x_{it}$

$\log x_{it} \sim U(R_t, S_t)$

independent across individuals and across time

R_t and S_t grow at rate g

g is productivity growth for all goods on s.s. path

If option 1, then $v_{it} = \log(P_{1t} x_{it}) + \beta E_t V_{1,t+1}$

Option 2: try to create a new good

Effort has utility cost k_U and probability of success k_π

Will collect compensation C_t while unemployed

C_t financed with tax on skilled workers

Let $V^{\sharp}_{t+1}$ be lifetime value of specializing in a new good

If option 2, then

$$v_{it} = \log(C_t) - k_U + k_\pi \beta E_t V^{\sharp}_{t+1} + (1 - k_\pi)\beta E_t V_{1,t+1}$$

Option 3: train to specialize in existing good

Probability of success π_{jt} = ratio of openings to applicants

Each period there are demand shocks that induce a fraction k_X of specialized goods to discontinue

$V_{jt} = \log Y_{jt} + \beta k_X E_t V_{j,t+1} + \beta(1 - k_X) E_t V_{1,t+1}$

If option 3, then

$v_{it} = \log(C_t) + \beta \pi_{jt} E_t V_{j,t+1} + \beta(1 - \pi_{jt}) E_t V_{1,t+1}$

$$v_{it} = \begin{cases} \log(P_{1t}x_{it}) + \beta E_t V_{1,t+1} & \text{option 1} \\ \log C_t + \beta \pi_{jt} E_t V_{j,t+1} + \beta(1-\pi_{jt}) E_t V_{1,t+1} & \text{option 2} \\ \log C_t - k_U + \beta k_\pi E_t V^{\sharp}_{t+1} + \beta(1-k_\pi) E_t V_{1,t+1} & \text{option 3} \end{cases}$$

Solution:

All individuals with $x_{it} > X^*_{1t}$ choose option 1

Let $\tilde{V}_{jt} = V_{jt} - V_{1t}$

$\log(P_{1t}X^*_{1t}) - \log C_t = -k_U + \beta k_\pi E_t \tilde{V}^\sharp_{t+1}$

$\log(P_{1t}X^*_{1t}) - \log C_t = \beta \pi_{jt} E_t \tilde{V}_{j,t+1}$

In equilibrium, π_{jt} adjusts to make both equations hold

If $\uparrow \tilde{V}_{j,t+1} \Rightarrow$ more people apply for $j \Rightarrow \downarrow \pi_{jt}$

5. Steady-state growth path

Along s.s. path, a fraction α_1 of before-tax national income goes to unskilled and $1 - \alpha_1$ to skilled

$$\frac{\sum_{j \in \mathcal{J}_{2t}} P_{jt} Q_{jt}}{P_{1t} Q_{1t}} = \frac{1-\alpha_1}{\alpha_1}$$

N_{1t} and X_{1t}^* determine Q_{1t}

$$Q_{1t} = N_{1t} \int_{X_{1t}^*}^{S_t} \frac{\exp z}{S_t - R_t} dz = N_{1t} \hat{X}_{1t}$$

This means average after-tax income of skilled and unemployment compensation are determined by n_{1t} and X_{1t}^{*}

$$Y_{st} = \frac{(1-\tau)\sum_{j\in\mathcal{J}_{2t}} P_{jt}Q_{jt}}{(1-n_{1t})N_t} = \left[\frac{(1-\tau)(1-\alpha_1)}{\alpha_1}\right]\left[\frac{P_{1t}n_{1t}\hat{X}_{1t}}{(1-n_{1t})}\right]$$

$$C_t = \left[\frac{\tau(1-\alpha_1)}{\alpha_1(1-h_{1t})}\right]P_{1t}\hat{X}_{1t}$$

If $n_{1t} = n_1^0$ and $h_{1t} = h_{10}$, then

$$Y_{st}/(P_{1t}\hat{X}_{1t}) = \frac{(1-\tau)(1-\alpha_1)n_1^0}{\alpha_1(1-n_1^0)}$$

$$C_t/(P_{1t}\hat{X}_{1t}) = \frac{\tau(1-\alpha_1)}{\alpha_1(1-h_1^0)}$$

If all specialized workers earn Y_{st}, then $\tilde{V}_{jt} = \tilde{V}_0$

h_{0t} = fraction of unemployed who try to create new goods

$h_{0t}(1-h_{1t})k_{\pi}n_{1t}N_t$ people will create new goods in $t+1$

If surviving goods add workers at rate n (population growth rate) then

$$n_{1,t+1} = n_{1t} + k_X(1-n_{1t}) - e^{-n}h_{0t}(1-h_{1t})k_{\pi}n_{1t}$$

(1) $n_{1,t+1} = n_{1t} + k_X(1 - n_{1t}) - e^{-n} h_{0t}(1 - h_{1t}) k_\pi n_{1t}$

(2) $\log(P_{1t} X_{1t}^*) - \log C_t = -k_U + \beta k_\pi E_t \tilde{V}_{t+1}$

(3) $\log(P_{1t} X_{1t}^*) - \log C_t = \beta \pi_t E_t \tilde{V}_{t+1}$

Proposition 3. For any $k_\pi, k_X, \alpha_1, \beta, \tau \in (0,1)$ and $n, k_U > 0$ there exists a unique value for $(X_{1t}^{*0}, n_{1t}^0, h_{0t}^0)$ for which (1)-(3) hold with $n_{1,t+1} = n_{1t}$

At this value, $E_t \tilde{V}_{t+1}$ is positive

Let n_1^0 be value determined by Proposition 3

Suppose $n_{jt_0}/\alpha_{jt_0} = (1 - n_1^0)/(1 - \alpha_1)\ \forall j$

$X_{jt_0} n_{jt_0} N_{t_0} = \int_0^{N_{t_0}} q_{ijt_0}^0 di = \bar{Q}_{jt}^0/2$

If $n_{jt} N_t$ individuals succeed in creating a new good j, they achieve the s.s. skill advantage:

$$\alpha_{jt} = n_{jt}(1 - n_1^0)/(1 - \alpha_1)\ j \in \mathcal{J}_{2t}^{\sharp}$$

Proposition 4. If $R_t, S_t, X_{jt}, q^0_{ijt}$ all grow at rate g and a fraction k_X of goods are discontinued each period, then $n_{1t}, h_{1t}, h_{0t}, P_{jt}/P_{1t}$ are constant

$\tilde{V}_{jt} = \tilde{V}^0 \ \forall j, t$

$X_{jt} n_{jt} N_t = \bar{Q}^0_{jt}/2$

$Q_{j,t+1} = e^{n+g} Q_{jt}$ for surviving goods

6. Adjustment to steady state

$$Q_{jt} = \min\{\zeta_{jt} X_{jt}^{0} N_{jt}, \bar{Q}_{jt}/2\}$$

$$\bar{q}_{ijt} = 2\chi_{jt} q_{ijt}^{0}$$

Individuals are on one of two steady-state paths

n_{1t} unskilled, $(1 - n_{1t})$ skilled

$$\bar{Q}_{jt} = \chi_{jt} H_t \bar{Q}_{jt}^{0}$$

$$H_t = 1 + \lambda_H (n_{1t} - n_1^0)$$

$$\lambda_H < 0$$

If goods j and k have same demand shock ($\chi_{jt} = \chi_{kt}$) and have capacity to produce at profit-maximizing level, they both have same output relative to steady state

From consumer F.O.C., it is always the case that

$$\frac{Y_{jt}}{P_{1t}} = \frac{Y_t^0 \xi_{jt}(1-n_1^0) Q_{jt}(\bar{Q}_{jt}-Q_{jt}) Q_{1t}^0}{(1-n_{1t})(Q_{jt}^0)^2(\bar{Q}_{1t}-Q_{1t})}$$

If $Q_{jt} = \bar{Q}_{jt}/2$,

$$\frac{Y_{jt}}{P_{1t}} = \frac{Y_t^0 (1-n_1^0) \chi_{jt}^2 H_t^2 Q_{1t}^0}{(1-n_{1t})(\bar{Q}_{1t}-Q_{1t})}$$

If goods have adequate capacity and same demand shock, workers have same income

$$\frac{Y_{jt}}{P_{1t}} = \frac{Y_t^0(1-n_1^0)\chi_{jt}^2 H_t^2 Q_{1t}^0}{(1-n_{1t})(\bar{Q}_{1t}-Q_{1t})}$$

Q_{1t} determined by $n_{1t}N_t$ (predetermined) and X_{1t}^* (from dynamic programming problem)

If Y_{jt} same for all j

$$\log X_{1t}^* - \log(C_t/P_{1t}) = -k_U + \beta k_\pi \tilde{V}_{t+1}$$

If $\chi_{j,t+s} = \zeta_{j,t+s} = 1\ \forall j, s \geq 0,$ dynamic system is

(1) $n_{1,t+1} = 1 - n^{\sharp}_{t+1} - n^{\natural}_{t+1}$

(2) $\tilde{V}_t = \log(Y_t/P_{1t}) - \log\tilde{X}_{1t} + \beta(1 - k_X)\tilde{V}_{t+1}$

(3) $\bar{n}_{t+1} = (1 - k_X)\bar{n}_t + n^{\sharp}_{t+1}$

(4) $Y_t/P_{1t} = \frac{Y^0_t(1-n^0_1)H^2_t Q^0_{1t}}{(1-n_{1t})(2H_t Q^0_{1t} - n_{1t}N_t\hat{X}_{1t})}$

(5) $C_t/P_{1t} = \frac{\tau(1-n_{1t})Y_t/P_{1t}}{n_{1t}(1-h_{1t})}$

(6) $\log X^*_{1t} - \log(C_t/P_{1t}) = -k_U + \beta k_\pi \tilde{V}_{t+1}$

(7) $\pi_t = \frac{e^n n^{\natural}_{t+1} - (1-k_X)(1-n_{1t})}{(1-h_{1t})(1-h_{0t})n_{1t}}$

(8) $n^{\sharp}_{t+1} = e^{-n}(1 - h_{1t})n_{1t}k_\pi - [n^{\natural}_{t+1} - e^{-n}(1 - k_X)(1 - n_{1t})]k_\pi/\pi_t$

(9) $n^{\natural}_{t+1} = H_{t+1}(1 - k_X)\bar{n}_t$

Demand or productivity shocks in periods $t_0, \ldots, t_0 + M$ will result in different equations for initial periods

Solution method: linearize around steady-state growth path and find rational-expectations solution

Fixed parameters

parameter	**meaning**
$\alpha_1 = 0.4$	steady-state expenditure share of good 1
$\tau = 0.02$	marginal tax rate
$\beta = 0.995$	discount rate
$k_U = 0.2$	utility cost of trying to create new good
$k_\pi = 0.25$	probability of successfully creating new good
$k_X = 0.02$	fraction of goods discontinued each period in steady state
$n = 0.0025$	population growth rate
$R_{t_0} = 1$	initial lowest log productivity of unskilled workers
$S_{t_0} = 2$	initial highest log productivity of unskilled workers

Steady-state magnitudes

variable	**meaning**
$n_1^0 = 0.4436$	fraction of population without skills
$\log X_{1t_0}^{*0} = 1.1154$	initial productivity threshold for unskilled workers to produce good 1
$1 - h_1^0 = 0.1154$	fraction of unskilled workers who are unemployed
$u^0 = 0.0512$	fraction of population who are unemployed
$\pi^0 = 0.2082$	probability of successfully becoming specialized in an existing good
$\tilde{V}^0 = 4.8032$	discounted lifetime log income differential between skilled and unskilled

9. Demand shocks

Example 9.1.
All variables start out at t_0 at steady-state values except that 25% of specialized goods have demand 10% below steady state ($\chi_{jt_0} = 0.9$).
Demand reverts to $\chi_{jt} = 1$ for all goods in $t_0 + 1$

$$Q_{jt} = \min\{\zeta_{jt} X^0_{jt} N_{jt}, \chi_{jt} H_t \bar{Q}^0_{jt}/2\}$$

Impacted goods lower production by 10%

Nonimpacted specialized goods don't change production

It is almost the case that:

- Impacted goods lower price by 10%
- Everything returns to normal in $t_0 + 1$


Fraction of workers without skills
0.03
0.02
0.01
0
0
2
4
6
8
10
12
14
16
18
Real GDP
0.01
0
-0.01
-0.02
0
2
4
6
8
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12
14
16
18

Why not exactly the case?
Lower taxes collected from skilled workers mean less unemployment compensation
Slight decrease in number of unskilled trying to create new goods
Slight increase in n_{1,t_0+1} above n_1^0 that gradually returns to steady state
Increased production of Q_{1t_0} relative to steady state is why real GDP does not quite fall by 1.5%

Example 9.2.

All variables start out at t_0 at steady-state values except that 25% of specialized goods have demand 10% above steady state ($\chi_{jt_0} = 1.1$).

Demand reverts to $\chi_{jt} = 1$ for all goods in $t_0 + 1$

$Q_{jt} = \min\{\zeta_{jt} X_{jt}^0 N_{jt}, \chi_{jt} H_t \bar{Q}_{jt}^0 / 2\}$

No specialized goods change production

It is approximately the case that:

Impacted goods raise price 20%

Everything back to normal in $t_0 + 1$

Fraction of workers without skills
0.03
0.02
0.01
0
0
2
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12
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16
18
Real GDP
0.01
0
-0.01
-0.02
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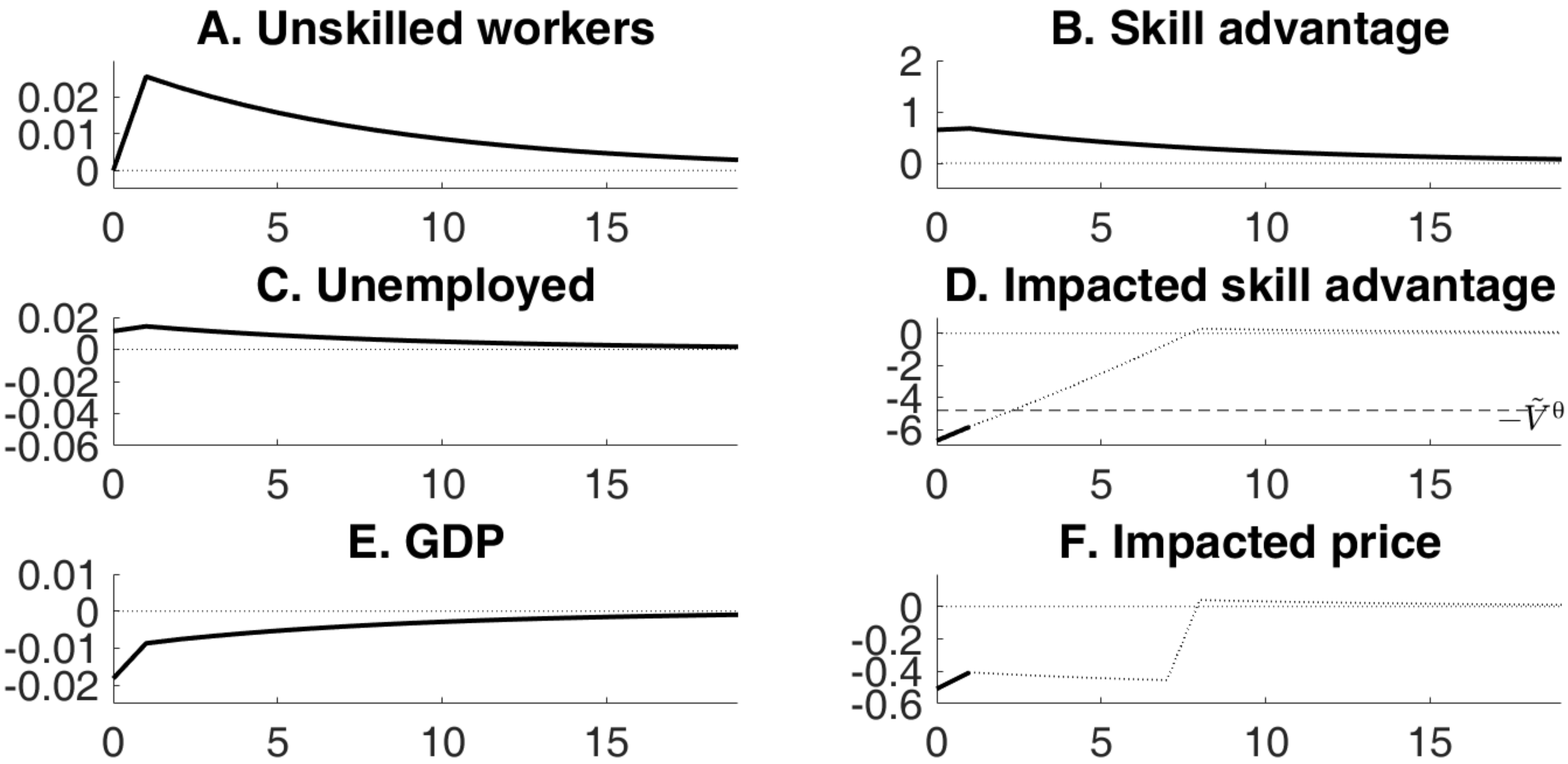
Why not exactly the case?
Higher taxes collected from skilled workers mean more unemployment compensation
Slight increase in number of unskilled trying to create new goods
Lower production of Q_{1t_0} relative to steady state is why real GDP actually falls

Example 9.3.

All variables start out at t_0 at steady-state values except that 5% of specialized goods have demand 50% below steady state ($\chi_{jt_0} = 0.5$).

Weak demand persists for 8 periods

Impacted goods would choose to discontinue production beginning in $t_0 + 1$



A. Unskilled workers
0.02
0.01
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0
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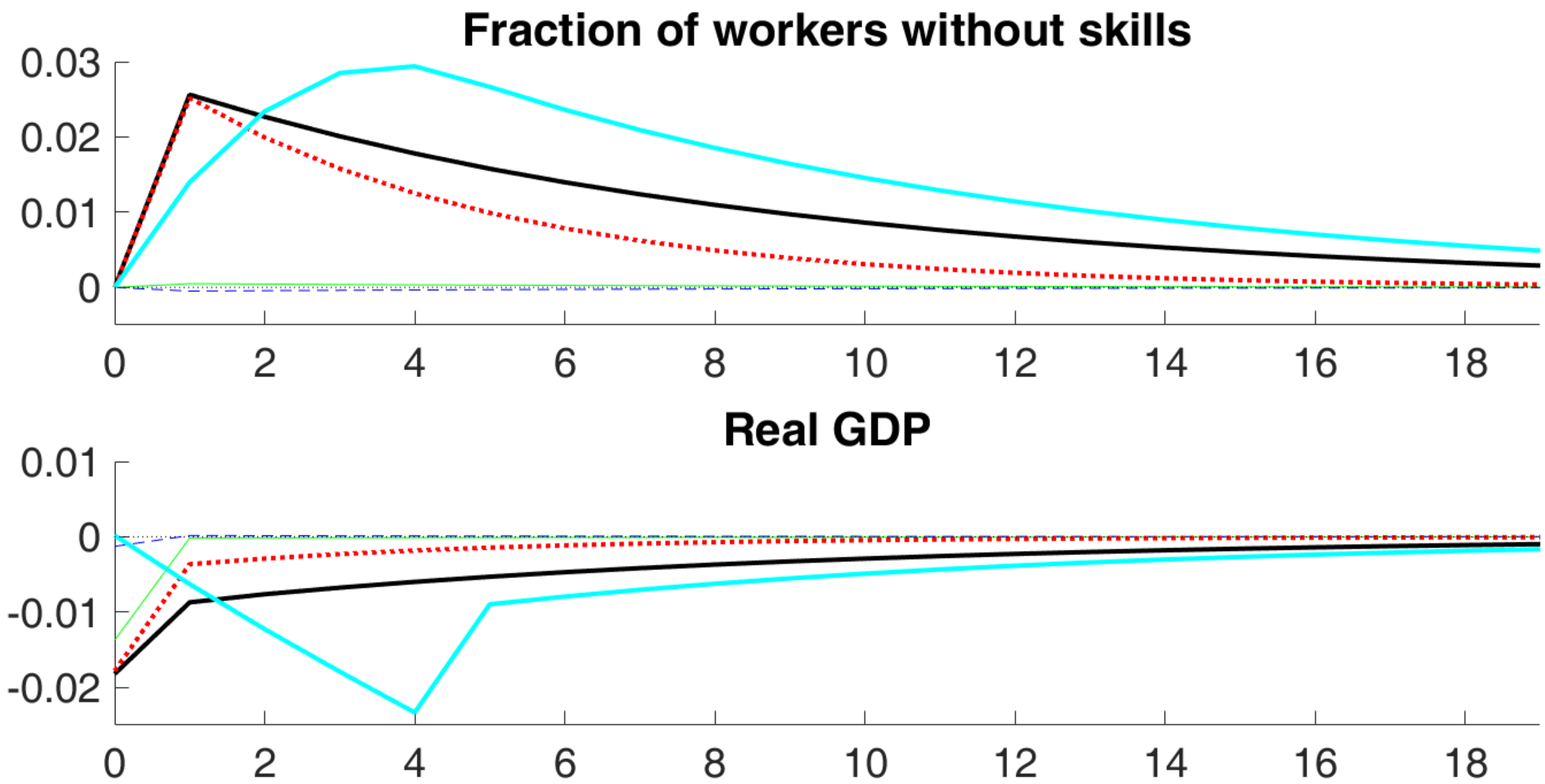
10
15
B. Skill advantage
2
1
0
0
5
10
15
C. Unemployed
0.02
0
-0.02
-0.04
-0.06
0
5
10
15
D. Impacted skill advantage
0
-2
-4
-6
$-\tilde{V}^{\theta}$
0
5
10
15
E. GDP
0.01
0
-0.01
-0.02
0
5
10
15
F. Impacted price
0
-0.2
-0.4
-0.6
0
5
10
15

Example 9.4.

Same as example 9.3 except with lower technological frictions; $k_\pi = 0.6$ (red) instead of baseline 0.25 (black)

Example 9.5.
25% of goods experience 10% decrease in demand that lasts for 5 periods
These goods remain in production but do no new hiring
If low demand also characterizes newly created goods in first 4 periods, there is less incentive to create new goods


Fraction of workers without skills
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Real GDP
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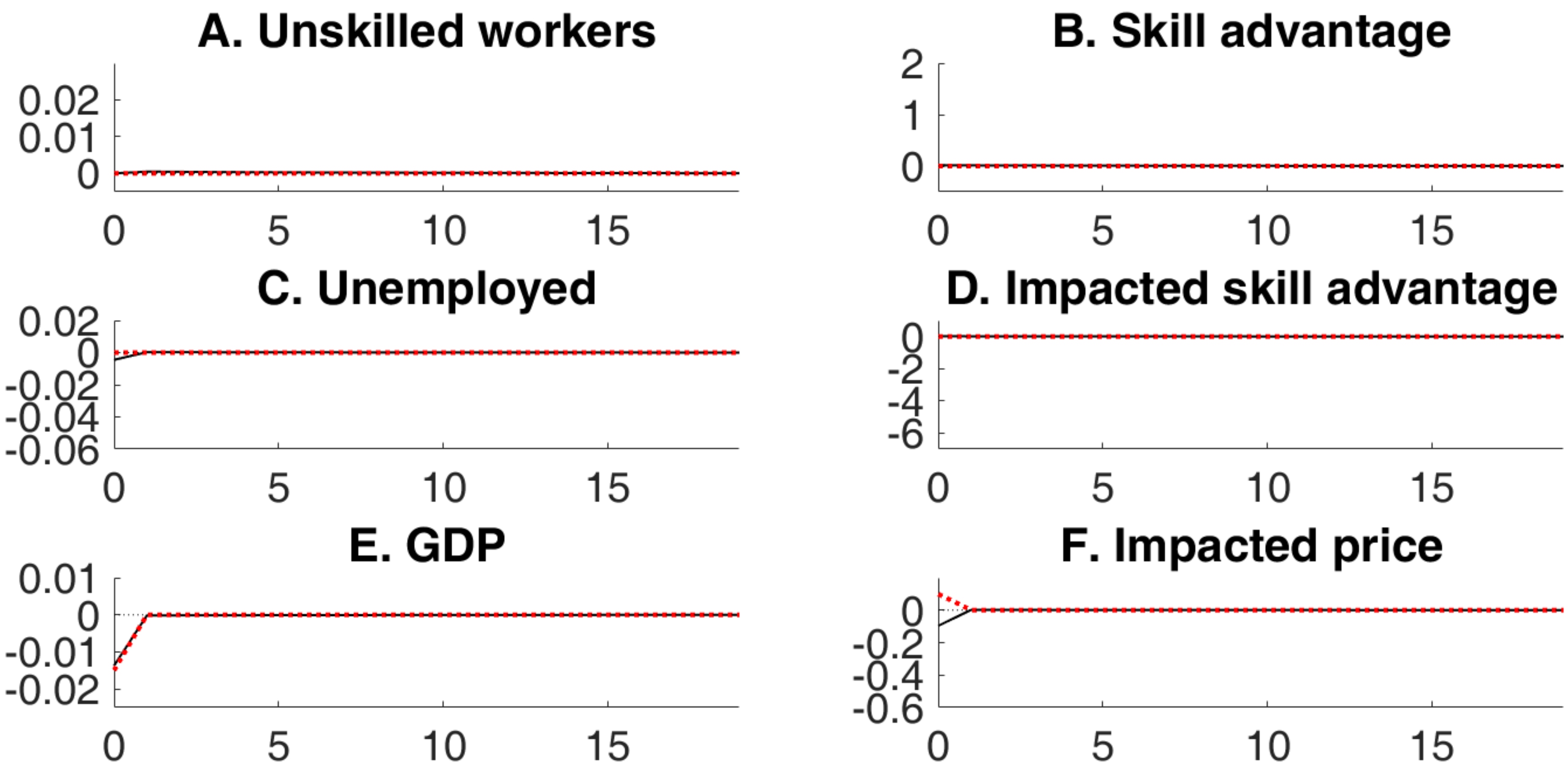
10. Productivity shocks

Example 10.1.
25% of goods experience 10% drop in productivity that lasts for one period ($\zeta_{jt_0} = 0.9$)
$Q_{jt} = \min\{\zeta_{jt} X^0_{jt} N_{jt}, \chi_{jt} H_t \bar{Q}^0_{jt}/2\}$
Impacted goods lower production 10%
and raise price 10%
Nonimpacted goods no change

Red: Example 10.1 (productivity shock); black: Example 9.1 (demand shock)



Demand shock:

$Q_{jt_0} \downarrow 10\%,\ N_{jt_0}$ unchanged, productivity $\downarrow$ 10%

Productivity shock:

$Q_{jt_0} \downarrow 10\%,\ N_{jt_0}$ unchanged, productivity $\downarrow$ 10%

Demand shock: $P_{jt_0} \downarrow$

Productivity shock: $P_{jt_0} \uparrow$

- Why would demand fall?
  - Some goods not wanted as much
- Why would productivity fall?
  - Natural disasters
- First happens all the time
- Can explain why it matters without relying on price rigidities